

# Profit-Function of Two Similar Warm Standby Navy Ship System Subject to Failure Due to Storm and Heavy Fog

Ashok Kumar Saini

BLJS College, Tosham (Bhiwani) Haryana, India  
 E-mail: drashokksaini2009@gmail.com

**Abstract**—In this paper we have taken failure due to storm, and heavy fog. When the main unit fails then warm standby system becomes operative. Failure due to heavy fog cannot occur simultaneously in both the units and after failure the unit undergoes Type-I or Type-II or Type-III repair facility immediately. Applying the regenerative point technique with renewal process theory the various reliability parameters MTSF, Availability, Busy period, Benefit-Function analysis have been evaluated.

## 1. INTRODUCTION

Many maritime disasters happen outside the realms of war. All ships, including those of the military, are vulnerable to problems from weather conditions, faulty design or heavy fog. Some of the disasters occurred in periods of conflict, although their losses were unrelated to any military action.

Year	Country	Description	Lives lost
256 BCE	Roman Republic	First Punic War – In the First Punic War, between the Roman Republic and Carthage, a Roman fleet that had just rescued a Roman army from Africa was caught in a Mediterranean storm. Rome may have lost more than 90,000 men.	90,000+
1588	Spain	Spanish Armada – On 8 August 1588, Philip II of Spain sent the Armada to invade England. Spain lost 15,000–20,000 soldiers and sailors, mainly in storms rather than battle.	15,000-20,000
1898	France	La Bourgogne – The passenger ship sank on 4 July 1898 after a collision in dense fog with the British ship Cromarty shire off Sable Island, Nova Scotia. La Bourgogne was carrying 730 passengers and crew, of whom 565 were lost.	565

Year	Country	Description	Lives lost
1854	United States	Arctic – a paddle steamer that sank 27 September 1854 off Cape Race, Newfoundland after colliding with the French iron screw steamship Vesta in fog. Of the 534 passengers and crew aboard, 350 were lost, including all 109 women and children.	350

In this paper we have taken failure due to storm, and heavy fog. When the main operative unit fails then warm standby system becomes operative. Failure due to heavy fog cannot occur simultaneously in both the units. After failure the unit undergoes repair facility of Type- I or Type- II by ordinary repairman, Type III or Type IV by multispecialty repairman immediately when failure due to storm and heavy fog. The repair is done on the basis of first fail first repaired.

## 2. ASSUMPTIONS

- $\lambda_1, \lambda_2, \lambda_3$  are constant failure rates when failure due to storm, failure due to heavy fog respectively. The CDF of repair time distribution of Type I, Type II and multispecialty repairmen Type-III, IV are  $G_1(t), G_2(t)$  and  $G_3(t), G_4(t)$ .
- The failure due to heavy fog is non-instantaneous and it cannot come simultaneously in both the units.
- The repair starts immediately after failure due to storm and failure due to heavy fog and works on the principle of first fail first repaired basis. The repair facility does no damage to the units and after repair units are as good as new.
- The switches are perfect and instantaneous.
- All random variables are mutually independent.
- When both the units fail, we give priority to operative unit for repair.
- Repairs are perfect and failure of a unit is detected immediately and perfectly.

8. The system is down when both the units are non-operative.

**Symbols for states of the System**

**Superscripts O, CS, SF, HFF,**

Operative, Warm Standby, failure due to storm, failure due to heavy fog respectively

**Subscripts nsf, sf, hff, ur, wr, uR**

No failure due to storm, failure due to storm, failure due to heavy fog, under repair, waiting for repair, under repair continued from previous state respectively

Up states – 0, 1, 2, 3, 10 ; Down states – 4, 5, 6, 7,8,9,11, regeneration point – 0,1,2, 3, 8, 9,10

**States of the System**

**0(O<sub>nsf</sub>, CS<sub>nsf</sub>)** One unit is operative and the other unit is warm standby and there is no failure due to storm of both the units.

**1(SF<sub>sf, urI</sub>, O<sub>nsf</sub>)** The operating unit failure due to storm is under repair immediately of Type- I and standby unit starts operating with no failure due to storm

**2(HFF<sub>hff, urII</sub>, O<sub>nsf</sub>)** The operative unit failure due to heavy fog and undergoes repair of Type II and the standby unit becomes operative with no failure due to storm

**3(HFF<sub>hff, urIII</sub>, O<sub>nsf</sub>)** The first unit failure due to heavy fog and under Type-III multispecialty repairman and the other unit is operative with no failure due to storm

**4(SF<sub>sf,urI</sub>, SF<sub>sf,wri</sub>)** The unit failed due to SF resulting from failure due to storm under repair of Type- I continued from state 1 and the other unit failed due to SF resulting from failure due to storm is waiting for repair of Type-I.

**5(SF<sub>sf,urI</sub>, HFF<sub>hff, wrII</sub>)** The unit failed due to SF resulting from failure due to storm is under repair of Type- I continued from state 1 and the other unit fails due to heavy fog is waiting for repair of Type- II.

**6(HFF<sub>hff, urII</sub>, SF<sub>sf, wri</sub>)** The operative unit failed due to heavy fog is under repair continues from state 2 of Type –II and the other unit failed due to SF resulting from failure due to storm is waiting under repair of Type-I.

**7(HFF<sub>hff, urII</sub>, SF<sub>sf, wrII</sub>)** The one unit failed due to heavy fog is continued to be under repair of Type II and the other unit failed due to SF resulting from failure due to storm is waiting for repair of Type-II.

**8(SF<sub>sf,urIII</sub>, HFF<sub>hff, wrII</sub>)** The one unit failure due to storm is under multispecialty repair of Type-III and the other unit failed due to heavy fog is waiting for repair of Type-II.

**9(SF<sub>sf,urIII</sub>, HFF<sub>hff, wri</sub>)** The one unit failure due to storm is under multispecialty repair of Type-III and the other unit failed due to heavy fog is waiting for repair of Type-I

**10(O<sub>nsf</sub> HFF<sub>hff, urIV</sub>)** The one unit is operative with no failure due to storm and warm standby unit fails due to heavy fog and undergoes repair of type IV.

**11(O<sub>nsf</sub> HFF<sub>hff, urIV</sub>)** The one unit is operative with no failure due to storm and warm standby unit fails due to heavy fog and repair of type IV continues from state 10.

**Transition Probabilities**

Simple probabilistic considerations yield the following expressions:

$$p_{01} = \lambda_1 / \lambda_1 + \lambda_2 + \lambda_3, p_{02} = \lambda_2 / \lambda_1 + \lambda_2 + \lambda_3, p_{0,10} = \lambda_3 / \lambda_1 + \lambda_2 + \lambda_3$$

$$p_{10} = pG_1^*(\lambda_1) + qG_2^*(\lambda_2),$$

$$p_{14} = p - pG_1^*(\lambda_1) = p_{11}^{(4)},$$

$$p_{15} = q - qG_1^*(\lambda_2) = p_{12}^{(5)},$$

$$p_{23} = pG_2^*(\lambda_1) + qG_2^*(\lambda_2),$$

$$p_{26} = p - pG_2^*(\lambda_1) = p_{29}^{(6)},$$

$$p_{27} = q - qG_2^*(\lambda_2) = p_{28}^{(7)},$$

$$p_{30} = p_{82} = p_{91} = 1,$$

$$p_{0,10} = pG_4^*(\lambda_1) + qG_4^*(\lambda_2),$$

$$p_{10,1} = p - pG_4^*(\lambda_1) = p_{10,1}^{(11)},$$

$$p_{10,2} = q - qG_4^*(\lambda_2) = p_{10,2}^{(11)} \tag{1}$$

We can easily verify that

$$p_{01} + p_{02} + p_{03} = 1,$$

$$p_{10} + p_{14} (=p_{11}^{(4)}) + p_{15} (=p_{12}^{(5)}) = 1,$$

$$p_{23} + p_{26} (=p_{29}^{(6)}) + p_{27} (=p_{28}^{(7)}) = 1$$

$$p_{30} = p_{82} = p_{91} = 1$$

$$p_{10,0} + p_{10,1}^{(11)} (=p_{10,1}) + p_{10,2}^{(12)} (=p_{10,2}) = 1 \tag{2}$$

And mean sojourn time is

$$\mu_0 = E(T) = \int_0^\infty P[T > t] dt$$

**3. MEAN TIME TO SYSTEM FAILURE**

$$\emptyset_0(t) = Q_{01}(t)[s] \emptyset_1(t) + Q_{02}(t)[s] \emptyset_2(t) + Q_{0,10}(t)[s] \emptyset_{10}(t)$$

$$\emptyset_1(t) = Q_{10}(t)[s] \emptyset_0(t) + Q_{14}(t) + Q_{15}(t)$$

$$\emptyset_2(t) = Q_{23}(t)[s] \emptyset_3(t) + Q_{26}(t) + Q_{27}(t), \emptyset_3(t) = Q_{30}(t)[s] \emptyset_0(t),$$

$$\emptyset_{10}(t) = Q_{10,0}(t)[s] \emptyset_{10}(t) + Q_{10,1}(t)[s] \emptyset_1(t) + Q_{10,2}(t)[s] \emptyset_2(t) \tag{3-6}$$

We can regard the failed state as absorbing

Taking Laplace-Stiljes transform of eq. (3-6) and solving for

$$\phi_0^*(s) = N_1(s) / D_1(s) \tag{7}$$

where

$$N_1(s) = \{Q_{01}^* + Q_{0,10}^* Q_{10,1}^*\} [Q_{14}^*(s) + Q_{15}^*(s)] + \{Q_{02}^* + Q_{0,10}^* Q_{10,2}^*\} [Q_{26}^*(s) + Q_{27}^*(s)]$$

$$D_1(s) = 1 - \{Q_{01}^* + Q_{0,10}^* Q_{10,1}^*\} Q_{10}^* - \{Q_{02}^* + Q_{0,10}^* Q_{10,2}^*\} Q_{23}^* - Q_{30}^* - Q_{0,10}^* Q_{10,0}^*$$

Making use of relations (1) & (2) it can be shown that  $\phi_0^*(0) = 1$ , which implies that  $\phi_0(t)$  is a proper distribution.

$$MTSF = E[T] = \frac{d}{ds} \phi_0^*(s) \Big|_{s=0}$$

$$= (D_1'(0) - N_1'(0)) / D_1(0)$$

$$= (\mu_0 + \mu_1 (p_{01} + p_{0,10} p_{10,1}) + (p_{02} + p_{0,10} p_{10,2})(\mu_2 + \mu_3) + \mu_{10} p_{0,10} / (1 - (p_{01} + p_{0,10} p_{10,1}) p_{10} - (p_{02} + p_{0,10} p_{10,2}) p_{23}) - p_{0,10} p_{10,0})$$

where

$$\mu_0 = \mu_{01} + \mu_{02} + \mu_{0,10}$$

$$\mu_1 = \mu_{10} + \mu_{11}^{(4)} + \mu_{12}^{(5)}$$

$$\mu_2 = \mu_{23} + \mu_{28}^{(7)} + \mu_{29}^{(6)}$$

$$\mu_{10} = \mu_{10,0} + \mu_{10,1} + \mu_{10,2}$$

#### 4. AVAILABILITY ANALYSIS

Let  $M_i(t)$  be the probability of the system having started from state  $i$  is up at time  $t$  without making any other regenerative state. By probabilistic arguments, we have

$$M_0(t) = e^{-\lambda_1 t} e^{-\lambda_2 t} e^{-\lambda_3 t},$$

$$M_1(t) = p G_1(t) e^{-\lambda_1 t}$$

$$M_2(t) = q G_2(t) e^{-\lambda_2 t}, M_3(t) = G_3(t),$$

$$M_{10}(t) = G_4(t) e^{-\lambda_3 t}$$

The point wise availability  $A_i(t)$  have the following recursive relations

$$A_0(t) = M_0(t) + q_{01}(t)[c]A_1(t) + q_{02}(t)[c]A_2(t) + q_{0,10}(t)[c]A_{10}(t)$$

$$A_1(t) = M_1(t) + q_{10}(t)[c]A_0(t) + q_{12}^{(5)}(t)[c]A_2(t) + q_{11}^{(4)}(t)[c]A_1(t),$$

$$A_2(t) = M_2(t) + q_{23}(t)[c]A_3(t) + q_{28}^{(7)}(t)[c]A_8(t) + q_{29}^{(6)}(t)[c]A_9(t)$$

$$A_3(t) = M_3(t) + q_{30}(t)[c]A_0(t),$$

$$A_8(t) = q_{82}(t)[c]A_2(t)$$

$$A_9(t) = q_{91}(t)[c]A_1(t),$$

$$A_{10}(t) = M_{10}(t) + q_{10,0}(t)[c]A_0(t) + q_{10,1}^{(11)}(t)[c]A_1(t) + q_{10,2}^{(11)}(t)[c]A_2(t) \tag{8-15}$$

Taking Laplace Transform of eq. (8-15) and solving for

$$\hat{A}_0(s)$$

$$\hat{A}_0(s) = N_2(s) / D_2(s) \tag{16}$$

where

$$N_2(s) = \{ \hat{q}_{0,10} \bar{M}_{10} + \bar{M}_0 \} [ \{ 1 - \hat{q}_{11}^{(4)} \} \{ 1 - \hat{q}_{28}^{(7)} \hat{q}_{82} \} - \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91} ] + \{ \hat{q}_{01} + \hat{q}_{0,10} \}$$

$$\hat{q}_{0,10} \hat{q}_{10,1}^{(11)} [ \bar{M}_1 \{ 1 - \hat{q}_{28}^{(7)} \hat{q}_{82} \} + \hat{q}_{12}^{(5)} \hat{q}_{23} \bar{M}_3 + \bar{M}_2 ] + \{ \hat{q}_{02} + \hat{q}_{0,10} \hat{q}_{10,2}^{(11)} [ \{ \hat{q}_{23} \bar{M}_3 \} \{ 1 - \hat{q}_{11}^{(4)} \} + \hat{q}_{29}^{(6)} \hat{q}_{91} \bar{M}_1 ]$$

$$D_2(s) = \{ 1 - \hat{q}_{11}^{(4)} \} \{ 1 - \hat{q}_{28}^{(7)} \hat{q}_{82} \} - \hat{q}_{12}^{(5)} \hat{q}_{29}^{(6)} \hat{q}_{91} - \{ \hat{q}_{01} + \hat{q}_{0,10} \hat{q}_{10,1}^{(11)} \} [ \hat{q}_{10} \{ 1 - \hat{q}_{28}^{(7)} \hat{q}_{82} \} + \hat{q}_{12}^{(5)} \hat{q}_{23} \hat{q}_{30} ] - \{ \hat{q}_{02} + \hat{q}_{0,10} \hat{q}_{10,2}^{(11)} \} [ \{ \hat{q}_{23} \hat{q}_{30} \} \{ 1 - \hat{q}_{11}^{(4)} \} + \hat{q}_{29}^{(6)} \hat{q}_{91} \hat{q}_{10} ]$$

(Omitting the arguments  $s$  for brevity)

The steady state availability

$$A_0 = \lim_{t \rightarrow \infty} [A_0(t)] = \lim_{s \rightarrow 0} [s \hat{A}_0(s)] = \lim_{s \rightarrow 0} \frac{s N_2(s)}{D_2(s)}$$

Using L' Hospital's rule, we get

$$A_0 = \lim_{s \rightarrow 0} \frac{N_x(s) + s N_x'(s)}{D_x(s)} = \frac{N_x(0)}{D_x(0)} \tag{17}$$

The expected up time of the system in (0,t] is  $\lambda_{ux}(t) =$

$$\int_0^t A_0(z) dz \text{ So that } \bar{\lambda}_{ux}(s) = \frac{\hat{A}_0(s)}{s} = \frac{N_x(s)}{s D_x(s)} \tag{18}$$

The expected down time of the system in (0,t] is  $\lambda_{dx}(t) = t -$

$$\lambda_{ux}(t) \text{ So that } \bar{\lambda}_{dx}(s) = \frac{1}{s^2} - \bar{\lambda}_{ux}(s) \tag{19}$$

Similarly, we can find out

1. The expected busy period of the server when there is failure due to storm, and heavy fog in (0,t]-R<sub>0</sub>
2. The expected number of visits by the repairman Type-I or Type-II for repairing the identical units in (0,t]-H<sub>0</sub>
3. The expected number of visits by the multispecialty repairman Type-III or Type-IV for repairing the identical units in (0,t]-W<sub>0</sub>, Y<sub>0</sub>.

**5. BENEFIT-FUNCTION**

The Benefit-Function analysis of the system considering mean up-time, expected busy period of the system under failure due to storm, and heavy fog, expected number of visits by the repairman for unit failure. The expected total Benefit-Function incurred in (0,t] is

$$C = \lim_{t \rightarrow \infty} (C(t)/t) = \lim_{s \rightarrow 0} (s^2 C(s)) = K_1 A_0 - K_2 R_0 - K_3 H_0 - K_4 W_0 - K_5 Y_0$$

where

K<sub>1</sub> - revenue per unit up-time, K<sub>2</sub> - cost per unit time for which the system is busy under repairing, K<sub>3</sub> - cost per visit by the repairman type- I or type- II for units repair,

K<sub>4</sub> - cost per visit by the multispecialty repairman Type- III for units repair,

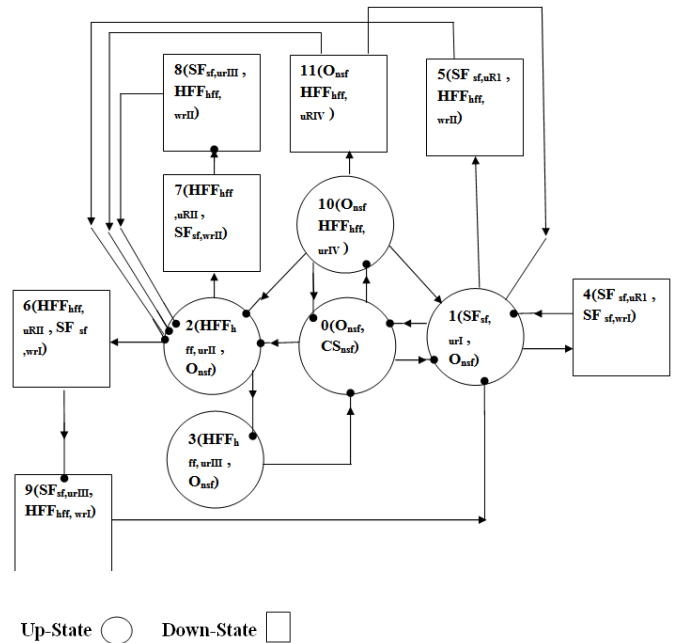
K<sub>5</sub> - cost per visit by the multispecialty repairman Type- IV for units repair

**6. CONCLUSION**

After studying the system, we have analyzed graphically that when the failure rate due to storm and due to heavy fog increases, the MTSF, steady state availability decreases and the Profit-function decreased as the failure increases.

**REFERENCES**

- [1] Dhillon, B.S. and Natesen, J, Stochastic Analysis of outdoor Power Systems in fluctuating environment, Microelectron. Reliab. ,1983; 23, 867-881.
- [2] Kan, Cheng, Reliability analysis of a system in a randomly changing environment, Acta Math. Appl. Sin. 1985, 2, pp.219-228.
- [3] Cao, Jinhua, Stochastic Behaviour of a Man Machine System operating under changing environment subject to a Markov Process with two states, Microelectron. Reliab. ,1989; 28, pp. 373-378.



**Fig. 1: The State Transition Diagram**

- [4] Barlow, R.E. and Proschan, F., Mathematical theory of Reliability, 1965; John Wiley, New York.
- [5] Gnedanke, B.V., Belyayar, Yu.K. and Soloyer , A.D. , Mathematical Methods of Reliability Theory, 1969 ; Academic Press, New York.